3- REPRESENTATION OF A CONTROL SYSTEM BY SIGNAL FLOW GRAPH

Signal flow graph is a graphical representation of algebraic equations. In this chapter, let us discuss the basic concepts related signal flow graph and also learn how to draw signal flow graphs.

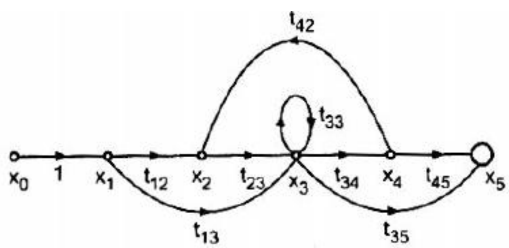
3.1- Basic Elements of Signal Flow Graph

Nodes and branches are the basic elements of signal flow graph.

- **Node** is a point which represents either a variable or a signal. There are three types of nodes (input node, output node and mixed node).

- **Input Node** − It is a node, which has only outgoing branches.
- **Output Node** − It is a node, which has only incoming branches.
- **Mixed Node** − It is a node, which has both incoming and outgoing branches.

3.2- *Important Terms In SFG :* Consider a signal flow graph shown below :



- Source Node : The node having only outgoing branches is known as source or input node. eg. X0 is source node.
- Sink Node : The node having only incoming branches is known as sink or output node.
- Chain Node : A node having incoming and outgoing branches is known as chain node. eg. X_1,x_2,x_3,x_4
- Forward Path : A path from the input to output node is defined as forward path.

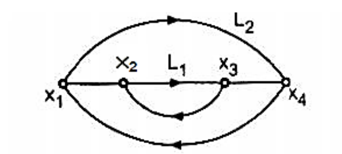
eg. $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5$ 1st forward path

$x_0 \rightarrow x_1 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5$ 2nd forward path

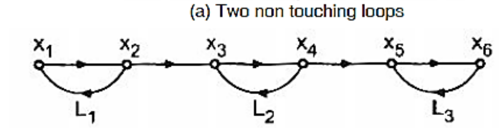
$x_0 \rightarrow x_1 \rightarrow x_3 \rightarrow x_5$ 3rd forward path

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_5$ 4th forward path

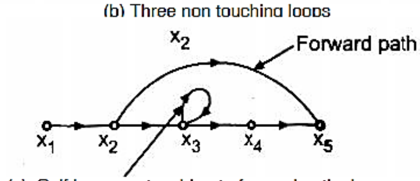
- Feedback Loop : A loop which originates and terminates at the same node is known as feedback path i.e. $x_2 \to x_3 \to x_4 \to x_2$ .No node is to be traced twice.
- Self Loop : A feedback loop consisting of only one node is called self loop. i.e. t_{33} at x_3 is self loop.
- Path Gain : The product of branch gains while going through a forward path is known as path gain i.e. path gain for path $x_0 \to x_1 \to x_2 \to x_3 \to x_4 \to x_5$ is, $t_{12}\, t_{23}\, t_{34}\, t_{45} \cdots$ This can also be called forward path gain.
- Non−touching Loops : If there is no node common in between the two or more loops, such loops are said to be non−touching loops



(a) Two non touching loops

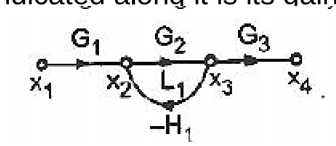


(b) Three non touching loops

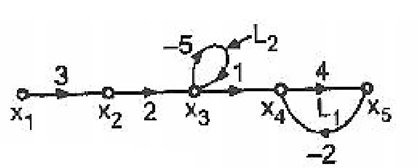


(c) Self loop non touching to forward path shown

- Loop Gain : The product of all the gains of the branches forming a loop is called as loop gain. For a self loop, gain indicated along it is its gain. Generally, such loop gains are denoted by 'L'.



In the above figure, there is one loop with gain L1 = G2 × − H1



Here there are two loops with gains

$L_1 = 4 \times -2 = -8$ and other self loop with $L_2 = -5$

3.3- METHODS TO OBTAIN SIGNAL FLOW GRAPH :

1- *From System Equations*

Steps :

1) Represent each variable by a separate node

2) Use the property that value of the variable represented by a node is an algebraic sum of all the signals entering at that node, to simulate the equations.

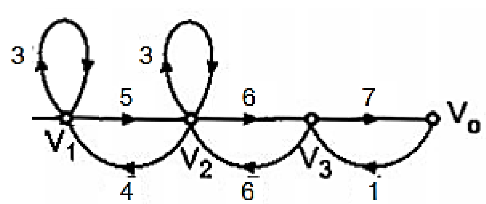
3) Coefficients of the variables in the equations are to be represented as the branch gains, joining the nodes in signal flow.

4) Show the input and output variables separately to complete signal flow graph.

Example :

Consider the following system equations :

$$V_1 = 3\,V_1 + 4\,V_2$$
$$V_2 = 5\,V_1 + 6\,V_3 + 3\,V_2$$
$$V_3 = 6\,V_2 + V_0$$
$$V_0 = 7\,V_3$$



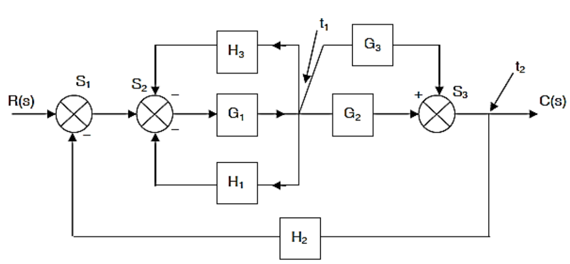
2- From given block diagram :

1) Name all the summing points and take off points in the block diagram.

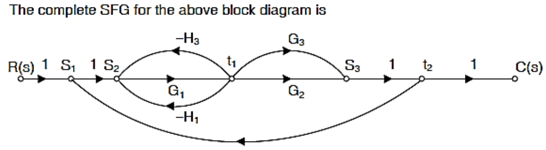
2) Represent each summing and take off point by a separate node in signal flow graph.

3) Connect them by the branches instead of blocks, indicating block transfer functions as the gain of the corresponding branches.

4) Show the input and output nodes separately if required to complete signal flow graph.



The complete SFG for the above block diagram is



3.4- MASON'S GAIN FORMULA

The overall transmittance (gain) can be determined by Mason's gain formula given below

$$\frac{\sum_{k=1} P_k \Delta_k}{\Delta}$$

The terms in the Mason's gain formula are explained below :

P_k - is the forward path transmittance of k_{th} path from a specified input node to an output node.

Δ - is the graph determinant which involves closed-loop transmittances and mutual interactions between non-touching loops.

Δ = 1 - [Sum of all individual loop transmittances] + [Sum of loop transmittance products of all possible pairs of ***NON-TOUCHING*** loops] - [Sum of loop transmittance products of all possible triplets of NON-TOUCHING loops] + [. ....] - [. ....]

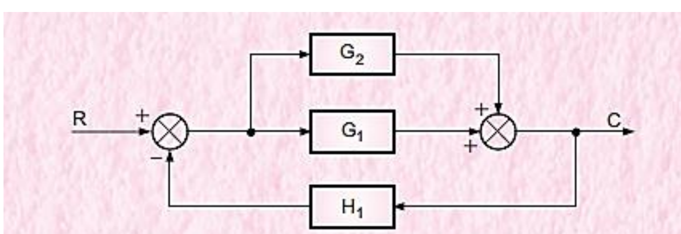
Δ_k - is the path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

The path factor Δ_k for the k_{th} path is equal to the value of the graph determinant of a signal flow graph which exists after ERASING the k_{th} path from the graph.

DRAWING SIGNAL FLOW GRAPH FROM A GIVEN BLOCK DIAGRAM

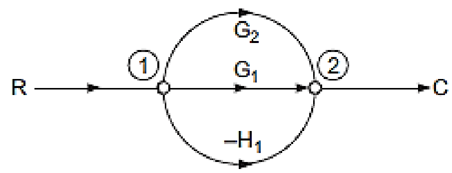
While drawing a signal flow graph from a given block diagram the adjacent ***summing points and take off points*** are represented by ***a node*** and the block ***transfer function*** is represented by a line joining the respective nodes. The direction of signal flow is indicated by an arrow on the line. However, in the direction of signal flow if, a takeoff point precedes a summing point then such points are represented by two separate nodes with a transmittance of unity between them.

Example 3.1. *Represent the block diagram given in Fig. below by signal flow graph and determine the overall transmittance relating C and R.*



Solution :-

Represent adjacent summing points, take off points before and after the block G_1 respectively by nodes (1) and (2). Join nodes **(1)** and (2) by lines indicating respective transmittance as per block diagram. The resulting signal flow graph is shown in Fig. below



Signal flow graph representation of block diagram

- The forward paths are $P_1 = G_1$ and $P_2 = G_2$
- The two closed loops are $L_1 = -G_1H_1$ and $L_2 = -G_2H_1$

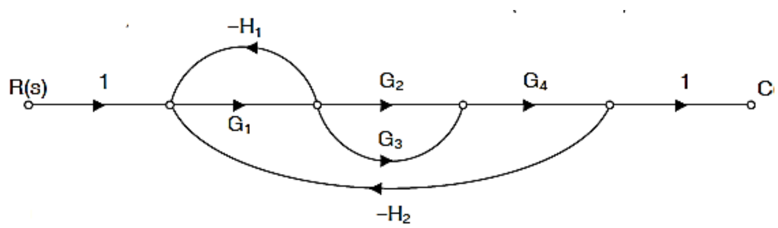
As both the loops L_1 and L_2 are touching both forward paths the two path factors are $\Delta_1 = \mathbf{1}$ and $\Delta_2 = \mathbf{1}$

- The graph determinants is $\Delta = 1 - (L_1 + L_2) = \mathbf{1} - (-G_1H_1 - G_2H_1) = \mathbf{1} + G_1H_1 + G_2H_1$

Applying Mason's gain formula the overall transmittance r elating C and R is determined below

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 .1 + G_2 .1}{1 + G_1H_1 + G_2H_1} = \frac{G_1 + G_2}{1 + G_1H_1 + G_2H_1}.$$

Example 3.2. Find C(s)/R(S) for the control system whose signal flow graph is shown below.



Solution : Number of forward paths = k = 2 By Mason's Gain formula,

$$\frac{\sum_{k=1} P_k\Delta_k}{\Delta} = \frac{P1\Delta1 + P2\Delta2}{\Delta}$$

- The forward paths are $P_1 = G_1 G_2 G_4$ All loops are touching $\Delta1 = 1$
  $P_2 = G_1 G_3 G_4$ All loops are touching $\Delta2 = 1$

- Individual feedback loops : $L_1 = -G_1 H_1$

$L_2 = -G_1 G_2 G_4 H_2$

$L_3 = -G_1 G_3 G_4 H_2$

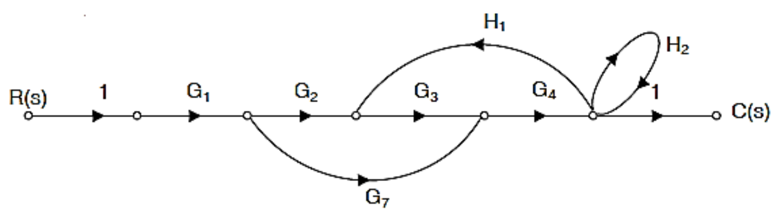
- The graph determinants is

$\Delta = 1 - [L_1 + L_2 + L_3]$ All loops are touching

$\Delta = 1 + G_1 H_1 + G_1 G_2 G_4 H_2 + G_1 G_3 G_4 H_2$

$$T.F = \frac{C(S)}{R(S)} = \frac{G1G2G4 + G1G3G4}{\Delta} = \frac{G1G2G4 + G1G3G4}{1 + G1H1 + G1G2G4H2 + G1G3G4H2}$$

Example 3.3. Determine the ratio of C (s) to R (S) for the control system whose signal flow graph is depicted in the picture below.



Solution : Number of forward paths = k = 2 By Mason's Gain formula,

$$\frac{\sum_{k=1} P_k \Delta_k}{\Delta} = \frac{P1\Delta1 + P2\Delta2}{\Delta}$$

- The forward paths are $P1 = G_1 G_2 G_3 G_4$ All loops are touching $\Delta_1 = 1$

$P_2 = G_1 G_4 G_7$ All loops are touching $\Delta_2 = 1$

- Individual feedback loops : $L_1 = G_3 G_4 H_1$

$L_2 = H_2$

- The graph determinants is

$\Delta = 1 - [L_1 + L_2] = 1 - G_3 G_4 H_1 - H_2$

$$T.F=\frac{C(S)}{R(S)}=\frac{P1\Delta1+P2\Delta2}{\Delta}=\frac{G1G2G3G4+G1G4G7}{1-G3G4H1-H2}$$

LMR(LAST MINUTE REVISION)

- The transfer function is defined as the ratio of Laplace transform of output to Laplace transform of input under assumption that all initial conditions are zero
- The stability of a time−invariant line system can be determined from the characteristic equation. Consequently, for continuous systems, if all the roots of the denominator have negative real parts, the system is stable.
- The system differential equation can be obtained from the transfer function by eplacing the s variable with d/dt.
- Transfer function is valid only for linear time invariant system.
- The value of ***s*** for which the system magnitude | G(s) | becomes infinity are called poles of G(s). When pole values are not repeated, such poles are called as simple poles. If repeated such poles are called multiple poles of order equal to the number of times they are repeated.
- The value of s for which the system magnitude | G(s) | becomes zero are called zeros of transfer function G(s). When they are not repeated, they are called simple zero, otherwise they are called multiple zeros.
- In Block diagram reduction, gain of the blocks in series gets multiplied whereas that of in parallel gets added or subtracted depending upon the sign of the summer.

Signal flow graph specifications :

i) The node having only outgoing branches is known as source or input node.
ii) The node having only incoming branches is known as sink or output node.
iii) A node having incoming and outgoing branches is known as chain node.
iv) A path from an input to an output node is defined as forward path.
v) A loop which originates and terminates on the same node is known as feedback path.
vi) A feedback loop consisting of only one node is called self loop.
vii) A self loop cannot appear while defining a forward path or feedback path as node containing it gents traced twice which is not allowed.

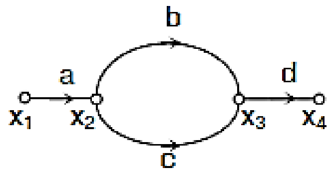
viii) The product of branch gains while going through a forward path is known as path gain. This can also be called forward path gain.

HOME WORK M

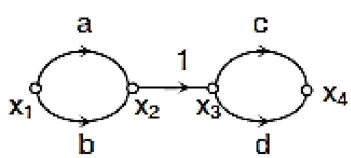
Q.1- Match List−I with List−II and select the correct answer using the codes given below

List−I (Signal flow graphs)

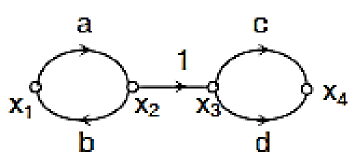
(a)



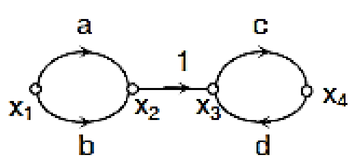
(b)



(c)

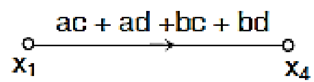


(d)

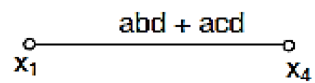


List−II (Simplification)

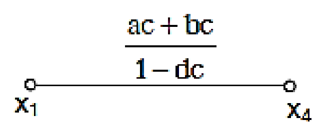
1.



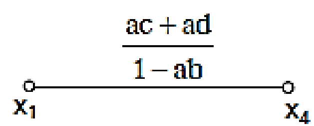
2.



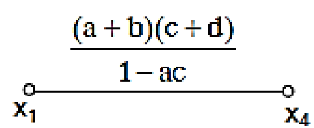
3.



4.



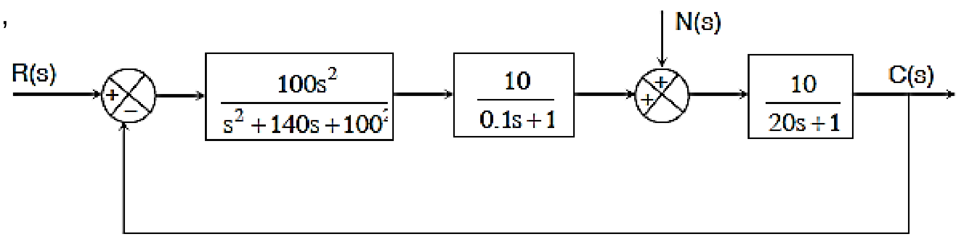
5.

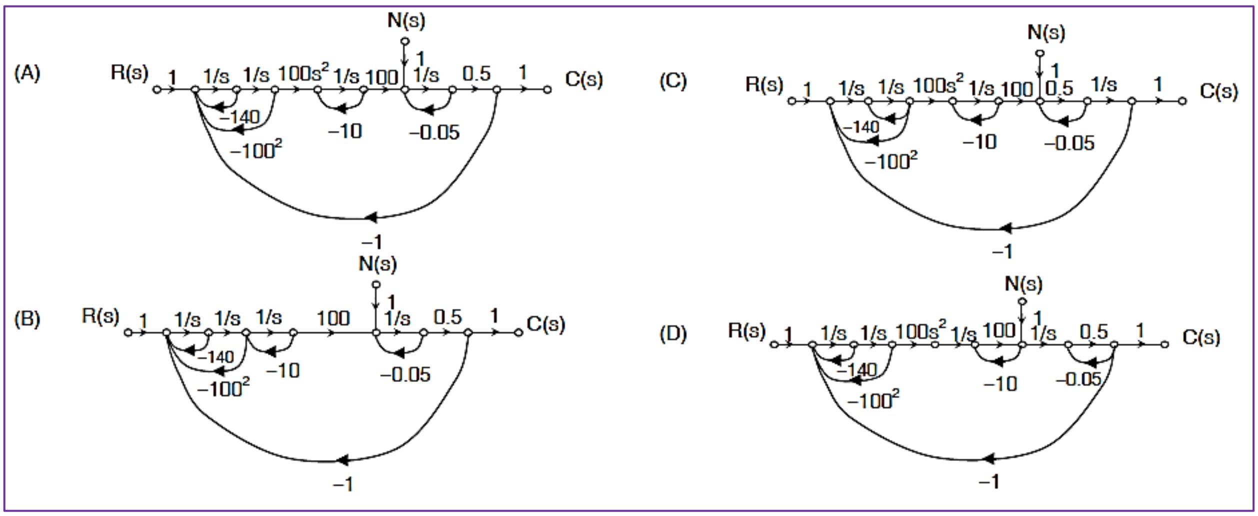


Codes :

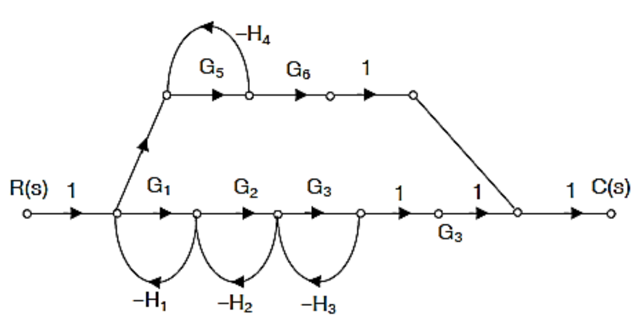
	A	B	C	D
(A)	2	1	5	4
(B)	3	4	2	1
(C)	2	1	4	3
(D)	1	2	3	5

Q.2- The figure below shows a block diagram of a control system. The signal flow graph for this system will be,

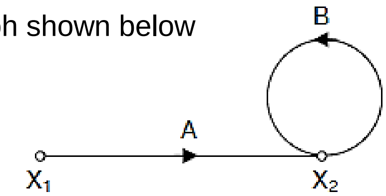




Q.3- In the below signal flow graph, there are ----------------- number of combinations of two non−touching loops.

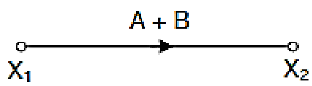


Q.4- The signal flow graph shown below

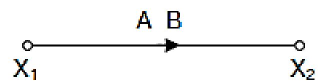


can be written as in ------------------.

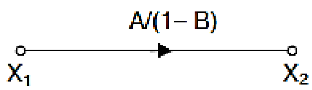
(A)



(B)



(C)



(D)